ANALYZING FIFO-MULTIPLEXING TANDEMS WITH NETWORK CALCULUS AND A TAILORED GRID SEARCH (SHORT PAPER)

Alexander Scheffler (RUB), Steffen Bondorf (RUB) and Jens Schmitt (TUK)
Overview

- Deterministic Network Calculus (DNC) Motivation and Basics
- Objective and Approaches
- Related Work
- GS
- Evaluation
DNC Motivation and Basics

- Theory of deterministic queueing systems [Cruz91]
  - Worst-case bounds such as delay and backlog

Analyzing FIFO-Multiplexing Tandems with Network Calculus and a Tailored Grid Search (Short Paper) | A. Scheffler, S. Bondorf and J. Schmitt
DNC Motivation and Basics (2)

- Can be used for certifying performance guarantees of cyber-physical systems, e.g., airplanes
- Can aid in ranking different network topologies and configurations
**DNC Motivation and Basics (3) [LeBoudec01]**

- **Arrival curve** \( \forall 0 \leq s \leq t : A(t) - A(t - s) \leq \alpha(s) \)

- **Service curve** \( \forall t \geq 0 : A'(t) \geq \inf_{0 \leq s \leq t} \{A(t - s) + \beta(s)\} := A \otimes \beta(t) \)

![Arrival and Service Curves Diagram]

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DNC Motivation and Basics (4) [LeBoudec01]

- **Concatenation of servers** \( \beta_1 \otimes \beta_2 = \beta_{1,2} \)
- **Output bound** \( \alpha'(t) = \alpha \otimes \beta(t) := \sup_{u \geq 0} \{ \alpha(t + u) - \beta(u) \} \)
- **Delay bound** \( hdev(\alpha, \beta) = \inf \{ d \geq 0 : (\alpha \otimes \beta)(-d) \leq 0 \} \)
DNC Motivation and Basics (5) [LeBoudec01]

- **FIFO left-over service curve**

\[
\beta_{f_1}^{1,o}(t, \theta) = [\beta(t) - \alpha_2(t - \theta)]^\uparrow \cdot 1_{\{t > \theta\}} \forall \theta \geq 0
\]

\[
[g(x)]^\uparrow = \sup_{0 \leq z \leq x} g(z)
\]

\[
1_{\{t > \theta\}} := \begin{cases} 
1, & t > \theta \\
0, & t \leq \theta 
\end{cases}
\]
Objective and Approaches

- LUDB[Bisti08], LUDB-FF [Scheffler21] optimizes these free parameters

New Approach

- Instead of optimizing these free parameters, we employ a robust grid search (GS) for a better tradeoff between accuracy and runtime
- We use GS to rank different network topologies

Find our code and dataset at

- https://github.com/NetCal/DNC
- https://github.com/alexscheffler/dataset-itic2022
Related Work

- **SFA-FIFO**
  - Server-by-server analysis
  - Compute at each server the residual service curve, convolve them
  - Each occurring $\theta$ set statically, $T + \frac{\sigma}{R}$
  - Simple left-over curve but local view
Related Work (2)

- **LUDB [Bisti08]**
  
  - Nested interference: A tandem has nested interference iff for every pair of flows either both flows do not have common servers or the path of one flow is completely included in the path of the other.
Related Work (3)

- **LUDB [Bisti08]**
  - “convolution before subtraction” for nested interference
  - Optimizes \((\theta_1, \ldots, \theta_{|F_x|})\) w.r.t. delay bound

\[\beta_{\text{foi}}^{1.0} = (\beta_{f_1}^{1.0} \ominus \theta_1 \alpha_1) \otimes (\beta_{f_3}^{1.0} \ominus \theta_3 \alpha_3)\]

\[\beta_{f_3}^{1.0} = \beta_2 \otimes (\beta_{f_2}^{1.0} \ominus \theta_2 \alpha_2)\]

\[\beta_{f_1}^{1.0} = \beta_1\]

\[\beta_{f_2}^{1.0} = \beta_3\]
GS

- Start with \( \Theta = (\theta_1, \ldots, \theta_{|F_x|}) = (0, \ldots, 0) \) resulting in delay bound \( d_{\text{start}} \)
GS (2)

- **Start with** $\Theta = (\theta_1, \ldots, \theta_{|F_x|}) = (0, \ldots, 0)$ **resulting in delay bound** $d^{\text{start}}$

- **Procedure**
  - Partition the search space: $|F_x|$-dimensional grid
  - Each point on the grid: delay bound $d$
  - Try, for each $\theta_i$, $g$ different values between $0$ and $d^{\text{start}}$ with $g \in \mathbb{N}_{>1}$
  - **Stepsize:** $sp := \frac{d^{\text{start}}}{g - 1}$
  - **Hence,** $\theta_i \in \left[0, \frac{1}{g-1}d^{\text{start}}, \frac{2}{g-1}d^{\text{start}}, \ldots, \frac{g-1}{g-1}d^{\text{start}} \right]$
GS (3)

- Procedure
  - Results in $O(g^{\lvert F_x \rvert})$, i.e., exponential in the number of crossflows
  - Check for each $\theta_i$, if $\theta_i \leq d^{\text{current}}$ holds, otherwise such a combination can be safely skipped

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Algorithm 2 Grid search on a nested tandem

Input $i, \Theta$ Flow, parameter combination
Output $\beta_{l.o.}$ Left-over service curve for flow $i$

1: procedure $GS(i, \Theta)$
2:     for $(\theta_i \leftarrow 0; \theta_i \leq d^{\text{current}}; \theta_i \leftarrow \theta_i + sp)$ do
3:         $\Theta(i) \leftarrow \theta_i$
4:     if $i = \lvert F_x \rvert$ then
5:         $\beta_{l.o.}^{\text{foi}} \leftarrow $ COMPUTELEFTOVERSERVICE($\text{foi}, \Theta$)
6:         $d \leftarrow \text{hdev}(\alpha_{\text{foi}}, \beta_{l.o.}^{\text{foi}})$  \(\triangleright\) horizontal deviation
7:     if $d < d^{\text{current}}$ then
8:         $d^{\text{current}} \leftarrow d$
9:         $\beta_{l.o.} \leftarrow \beta_{l.o.}^{\text{foi}}$
10:    else
11:        $GS(i + 1, \Theta)$
12: return $\beta_{l.o.}$
```
GS (4)

- $|F_x| = 3$, $g = 4$, $\theta_i \in \left[0, \frac{1}{3} d_{\text{start}}^{\text{start}}, \frac{2}{3} d_{\text{start}}^{\text{start}}, d_{\text{start}}^{\text{start}}\right]$

- 1st iteration: $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, 0) \rightarrow d = d_{\text{start}}^{\text{start}} = d_{\text{current}}^{\text{current}}$

- 2nd iteration: $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, \frac{1}{3} d_{\text{start}}^{\text{start}}) \rightarrow d$, update $d_{\text{current}}^{\text{current}}$ if $d < d_{\text{current}}^{\text{current}}$

- 3rd iteration: $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, \frac{2}{3} d_{\text{start}}^{\text{start}}) \rightarrow d$, update $d_{\text{current}}^{\text{current}}$ if $d < d_{\text{current}}^{\text{current}}$

- 4th iteration: $\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, d_{\text{start}}^{\text{start}}) \rightarrow d$, update $d_{\text{current}}^{\text{current}}$ if $d < d_{\text{current}}^{\text{current}}$
GS (5)

- \(|F_x| = 3\), \(g = 4\), \(\theta_i \in \left[0, \frac{1}{3}d^{\text{start}}, \frac{2}{3}d^{\text{start}}, d^{\text{start}}\right]\)
- **4th iteration:** \(\Theta = (\theta_1, \theta_2, \theta_3) = (0, 0, d^{\text{start}}) \rightarrow d\), **update** \(d^{\text{current}}\) **if** \(d < d^{\text{current}}\)
- **5th iteration:** \(\Theta = (\theta_1, \theta_2, \theta_3) = (0, \frac{1}{3}d^{\text{start}}, 0) \rightarrow d\), **update** \(d^{\text{current}}\) **if** \(d < d^{\text{current}}\)
- **...**
- **64th iteration:** \(\Theta = (\theta_1, \theta_2, \theta_3) = (d^{\text{start}}, d^{\text{start}}, d^{\text{start}}) \rightarrow d\), **update** \(d^{\text{current}}\) **if** \(d < d^{\text{current}}\)
GS (6)

- **Lemma 1:** Let $g_1, g_2 \in \mathbb{N}_{\geq 1}$ with $k \cdot (g_1 - 1) = g_2 - 1$, for $k \in \mathbb{N}$. Then,
  \[
  \text{delay}(\text{GS-}g_1) \geq \text{delay}(\text{GS-}g_2) \text{ holds.}
  \]

- **Proof:**
  - **GS-}g_1:** Each $\theta_i \in \left[0, \frac{1}{g_1 - 1} d_{\text{start}}, \frac{2}{g_1 - 1} d_{\text{start}}, \ldots, \frac{g_1 - 1}{g_1 - 1} d_{\text{start}}\right]$
  - **GS-}g_2:** Each $\theta_i \in \left[0, \frac{1}{g_2 - 1} d_{\text{start}}, \frac{2}{g_2 - 1} d_{\text{start}}, \ldots, \frac{g_2 - 1}{g_2 - 1} d_{\text{start}}\right]$

  \[
  k \cdot (g_1 - 1) = g_2 - 1 \quad \Rightarrow \quad \frac{1}{k (g_1 - 1)} d_{\text{start}}, \frac{2}{k (g_1 - 1)} d_{\text{start}}, \ldots, \frac{k \cdot (g_1 - 1)}{g_1 - 1} d_{\text{start}}
  \]

  \[
  \theta_i(\text{GS-}g_1) = \frac{a}{g_1 - 1} d_{\text{start}} \quad \Rightarrow \quad \theta_i(\text{GS-}g_2) = \frac{b}{g_2 - 1} d_{\text{start}} \quad \text{with} \quad a \in \mathbb{N} \cap [0, g_1 - 1], \quad b = a \cdot k \in \mathbb{N} \cap [0, g_2 - 1]
  \]
Evaluation

- **Setup**
  - 2086 unique nested tandems
  - Arrival curves set to token bucket $\gamma_{p,\sigma} = \gamma_{1,1}$
  - Service curve set to rate latency $\beta_{R,T}$ with $T = 0$ and $R$ set to achieve a utilization of each server of 95%

- **Goal**
  - Find reasonable value of g (GS parameter) s.t. the ranking of the different tandems is close to the ranking with the LUDB analysis
Evaluation (2)

- Ranking deviation compared to LUDB

![Graph showing order deviation compared to LUDB for different tandem lengths.](image-url)
Evaluation (3)

- **Average analysis runtime**

![Average Analysis Runtime Graph](graph.png)

- **Legend:**
  - SFA–FIFO
  - GS–2
  - GS–3
  - GS–4
  - LUDB–FF

- **Axes:**
  - **X-axis:** Tandem Length [Server]
  - **Y-axis:** Average Runtime [s]

- **Data Points:**
  - For different tandem lengths and server configurations, the average analysis runtime is plotted, showing how the runtime increases with tandem length and server configuration.
Evaluation (4)

- Delay bound deviation of the top-ranked tandems

![Graph showing delay bound deviation vs tandem length for different tandems and a comparison with FIFO.]
Conclusion

• Exploration of network design space w.r.t. delay bounds in FIFO-Multiplexing tandems

• Benchmark several NC analyses for this task
  • SFA-FIFO’s bounds too coarse: Ranking deviation of up to 93% compared to LUDB-FF
  • New analysis GS:
    • Worst-Case order deviation of no more than 41% while GS is considerably faster than LUDB-FF
    • Precision and runtime can be improved with a parameter
    • g of 2-4 most suitable for striking a reasonable balance between quality of ranking and runtime
Thanks for your attention!

Questions?
References


